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Self-consistent renormalisation in thermofield dynamics

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Abstract. From the consideration of quasiparticles in thermal situations, we derive a universal expression of the self-consistent renormalisation condition in thermofield dynamics. This condition is valid even when the renormalised energy becomes time dependent, and provides us with four independent self-consistent equations for four real parameters, i.e. renormalised energy, dissipative coefficient, number density and a new degree of freedom χ, which is a phase in thermal doublet space.

1. Introduction

In quantum field theory the concept of 'quasiparticle' plays a significant role. The Heisenberg field, ψ_{H}, which is controlled by a fundamental field equation, is realised in observation through a set of quasiparticle fields, ψ, which satisfy free field equations (i.e. linear homogeneous equations), say

$$\Lambda(\partial)\psi = 0. \tag{1.1}$$

The state vector space is associated with the quasiparticles. We call the expression of ψ_{H}, in terms of ψ, the dynamical map and write it as $\psi_{\mathrm{H}}[\psi]$. In the usual quantum field theory, the dynamical map is the Haag expansion in which the quasiparticle field is the incoming (or outgoing) field.

Thermofield dynamics (TFD), which is a quantum field theory with thermal degrees of freedom, inherited the concept of quasiparticle from the usual quantum field theory. In TFD every degree of freedom is doubled (thermal doublet). The freedom of how the thermal doublet components of the quasiparticle mix (ψ^{μ}, $\bar{\psi}^{\mu}$, $\mu = 1, 2$) appears to be the thermal freedom. Since the structure has been summarised in many papers, it will not be repeated here. (For the equilibrium TFD see, for example, [1, 2]. For the non-equilibrium TFD see, for example, [3, 4].) It is due to the negative energy of the tilde quasiparticle that the energy in TFD is not lower bounded. This makes practically every quasiparticle unstable [4, 5] (i.e. dissipative) in TFD. Thus, the study of the nature of the quasiparticle is significant in the development of TFD [3-6].

In TFD, both the Heisenberg equation and the quasiparticle free equation are thermal doublet equations:

$$\Lambda(\partial)^{\mu\nu}\psi_{\mathrm{H}}^{\nu} = J[\psi_{\mathrm{H}}]^{\mu} \tag{1.2}$$

$$\Lambda(\partial)^{\mu\nu}\psi^{\nu} = 0. \tag{1.3}$$

When we use ψ^{μ} and $\bar{\psi}^{\mu}$ for the free fields in the interaction representation, we can identify [3, 4] the interaction Hamiltonian, $\hat{H}_{\mathrm{int}}(t)$, once a model Lagrangian is given.

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Following the general steps of the interaction representation, we introduce the time-evolution operator $\hat{V}(t, t_0)$:

$$i \frac{d}{dt} \hat{V}(t, t_0) = \hat{H}_{int}(t) \hat{V}(t, t_0) \tag{1.4}$$

in which t_0 is the initial time: $\hat{V}(t_0, t_0) = 1$. We usually consider the limit $t_0 \to -\infty$. It is convenient to define also

$$\hat{S} = \hat{V}(\infty, -\infty). \tag{1.5}$$

When a time-dependent system approaches an equilibrium state at $t = \infty$, a precise measurement of the single-particle state is possible only at equilibrium. (The meaning of time-dependent properties of quasiparticles during time change of the non-equilibrium system is not yet clear.) Thus, we become more interested in the dynamical map of $\hat{S}\psi_H^\mu$ and $\hat{S}\bar{\psi}_H^\mu$ rather than ψ_H^μ and $\bar{\psi}_H^\mu$ themselves. The dynamical map of $\hat{S}\psi_H^\mu$ can be put in the form:

$$\hat{S}\psi_H(x)^\mu = Z\left(t, \frac{1}{i}\vec{\nabla}\right)^{\mu\nu} \psi(x)^\nu + \ldots \tag{1.6a}$$

$$\hat{S}\bar{\psi}_H(x)^\mu = \bar{\psi}(x)^\nu \bar{Z}\left(t, -\frac{1}{i}\overleftarrow{\nabla}\right)^{\nu\mu} + \ldots \tag{1.6b}$$

where the dots stand for the higher-order normal products of quasiparticle fields. To avoid complication in use of symbols, we use Z and $\bar{Z}$ rather than the usual symbol $Z^{1/2}$ in (1.6).

From the dynamical map (1.6), we are naturally led to the renormalisation condition (see explicitly (3.8)) corresponding to the mass (or energy) renormalisation in the usual quantum field theory. In TFD, this condition is a 2×2 thermal matrix equation and therefore should provide us with more information than the energy. Indeed, in studies of simple cases of a reservoir model [3] and a one-loop calculation [4, 7], the renormalisation condition also led us to a differential equation for the particle numbers. However, a universal expression of the renormalisation condition in time-dependent thermal situations is not known and has not even been attempted so far. The purpose of this paper is to give this precise renormalisation condition and to report our results derived from it.

2. The quasiparticles

In this paper we consider only type-1 fields (i.e. fields without antiparticles). According to [3, 4], in which the canonical formalism of quasiparticle fields ψ^μ and $\bar{\psi}^\mu$ has been formulated, ψ^μ and $\bar{\psi}^\mu$ have the forms

$$\psi(x)^\mu = \int \frac{d^3k}{(2\pi)^{3/2}} \exp(i\boldsymbol{k} \cdot \boldsymbol{x}) a_{\boldsymbol{k}}(t)^\mu \tag{2.1a}$$

$$\bar{\psi}(x)^\mu = \int \frac{d^3k}{(2\pi)^{3/2}} \exp(-i\boldsymbol{k} \cdot \boldsymbol{x}) \bar{a}_{\boldsymbol{k}}(t)^\mu. \tag{2.1b}$$

Here $a_{\boldsymbol{k}}(t)^\mu$ and $\bar{a}_{\boldsymbol{k}}(t)^\mu$ satisfy the equal-time canonical commutation relation:

$$[a_{\boldsymbol{k}}(t)^\mu, \bar{a}_{\boldsymbol{l}}(t)^\nu]_\sigma = \delta^{\mu\nu}\delta(\boldsymbol{k} - \boldsymbol{l}) \tag{2.2}$$

with $\sigma = +1$ or -1 for bosons or fermions.

We now introduce [4] the quasiparticle creation and annihilation operators $(\xi_k, \xi_k^\dagger, \tilde{\xi}_k, \tilde{\xi}_k^\dagger)$, which satisfy

$$[\xi_k, \xi_l^\dagger]_\sigma = [\tilde{\xi}_k, \tilde{\xi}_l^\dagger]_\sigma = \delta(\boldsymbol{k} - \boldsymbol{l}). \tag{2.3}$$

The operators $\tilde{\xi}$ and $\tilde{\xi}^\dagger$ σ-commute with ξ and $\xi^\dagger$. The thermal ket and bra vacua, $|0\rangle$ and $\langle 0|$, are annihilated in the following way:

$$\xi_k|0\rangle = \tilde{\xi}_k|0\rangle = 0 \tag{2.4a}$$

$$\langle 0|\xi_k^\dagger = \langle 0|\tilde{\xi}_k^\dagger = 0. \tag{2.4b}$$

The thermal doublets ξ^μ and $\bar{\xi}^\mu$ are defined by

$$\xi^1 = \xi \qquad \xi^2 = \tilde{\xi}^\dagger \tag{2.5a}$$

$$\bar{\xi}^1 = \xi^\dagger \qquad \bar{\xi}^2 = -\sigma\tilde{\xi}. \tag{2.5b}$$

Note that these creation and annihilation operators are independent of t and that the state vector space of TFD is generated by their cyclic operations on $|0\rangle$ and $\langle 0|$.

For the canonical properties (2.2) and (2.3) to hold simultaneously, $a(t)^\mu$ and $\bar{a}(t)^\mu$ must be related to ξ^μ and $\bar{\xi}^\mu$ through a time-dependent Bogoliubov transformation:

$$a_k(t)^\mu = \mathscr{B}^{-1}(t, \boldsymbol{k})^{\mu\nu}\xi_k^\nu \tag{2.6a}$$

$$\bar{a}_k(t)^\mu = \bar{\xi}_k^\nu \mathscr{B}(t, \boldsymbol{k})^{\nu\mu}. \tag{2.6b}$$

The 2×2 matrix $\mathscr{B}$ is restricted by imposing a condition arising from the tilde conjugation rules [1, 3, 4], i.e.

$$[a(t)^\mu]^\sim = -\mathrm{i}\sigma\tau_2^{\mu\nu}\bar{a}(t)^\nu \tag{2.7a}$$

$$[\bar{a}(t)^\mu]^\sim = -\mathrm{i}a(t)^\nu\tau_2^{\nu\mu} \tag{2.7b}$$

and the same ones for ξ. Here τ_i $(i = 1\text{–}3)$ are the Pauli matrices. Under this restriction, $\mathscr{B}$ can be quite generally factorised into a diagonal matrix $E(t)$ and another time-dependent Bogoliubov matrix $B(t)$:

$$\mathscr{B}(t, \boldsymbol{k}) = E^{-1}(t, \boldsymbol{k})B(t, \boldsymbol{k}). \tag{2.8}$$

The matrix $E(t)$ is expressed by two real parameters ω and κ which we identify as fully renormalised energy and dissipative coefficient, respectively:

$$E(t, \boldsymbol{k}) = \exp\left(-\mathrm{i}\int_{t_0}^{t} \mathrm{d}s[\omega(s, \boldsymbol{k}) - \mathrm{i}\kappa(s, \boldsymbol{k})\tau_3]\right). \tag{2.9}$$

The matrix $B(t, \boldsymbol{k})$ is a function of the particle number $n(t, \boldsymbol{k})$ which is defined as

$$n(t, \boldsymbol{k}) = \langle 0|\bar{a}_k(t)^1 a_k(t)^1|0\rangle. \tag{2.10}$$

The $B(t, \boldsymbol{k})$ depends not only on $n(t, \boldsymbol{k})$ but also on the other free parameter (see [3, 4]). Throughout this paper, we use the simplest expression:

$$B(t, \boldsymbol{k}) = \begin{bmatrix} 1 + \sigma n(t, \boldsymbol{k}) & -n(t, \boldsymbol{k}) \\ -\sigma & 1 \end{bmatrix}. \tag{2.11}$$

It is a matter of convenience when one defines the time-dependent quasiparticle operators [3, 4] $\xi(t)^\mu$ and $\bar{\xi}(t)^\mu$ by combining $E(t)$ with ξ^μ and $\bar{\xi}^\mu$:

$$\xi(t)^\mu = E(t)^{\mu\nu}\xi^\nu \tag{2.12a}$$

$$\bar{\xi}(t)^\mu = \bar{\xi}^\nu E^{-1}(t)^{\nu\mu}. \tag{2.12b}$$

With this definition, $E(t)$ may be called the wavefunction of the quasiparticles. As (2.6) imply, $a(t)^\mu$ and $\bar{a}(t)^\mu$ are related to the quasiparticle operators through the time-dependent Bogoliubov transformation:

$$a(t)^\mu = B^{-1}(t)^{\mu\nu}\xi(t)^\nu = B^{-1}(t)^{\mu\sigma}E(t)^{\sigma\nu}\xi^\nu \tag{2.13a}$$

$$\bar{a}(t)^\mu = \xi(t)^\nu B(t)^{\nu\mu} = \bar{\xi}^\nu E^{-1}(t)^{\nu\sigma}B(t)^{\sigma\mu}. \tag{2.13b}$$

The important observation here is that $\langle 0|T|\xi(t_1)^\mu\bar{\xi}(t_2)^\nu]|0\rangle$ never diverges for $|t_1-t_2|\to\infty$ as long as κ is positive in (2.9). This is true also for multipoint T-product Green functions consisting of a^μ and $\bar{a}^\mu$, or of ψ^μ and $\bar{\psi}^\mu$. Any Green function other than the T-product ones diverges at infinite separation of time difference.

The expressions in (2.1), together with (2.9), (2.11) and (2.13), lead to the following form of the operator $\Lambda(\partial)$ in (1.3) for the quasiparticle free equation:

$$\Lambda(t,-\partial)^{\mu\nu} = \left[\mathrm{i}\frac{\partial}{\partial t} - \omega\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right) + \mathrm{i}P\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right)\right]^{\mu\nu}. \tag{2.14}$$

Here

$$P\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right) = \kappa\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right)A\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right) + \sigma\dot{n}\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right)^{\tau_0} \tag{2.15}$$

$$A,\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right) = B^{-1}\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right)\tau_3 B\left(t,\frac{1}{\mathrm{i}}\vec{\nabla}\right) = \begin{bmatrix} 1+2\sigma n & -2n \\ 2\sigma(1+\sigma n) & -(1+2\sigma n)\end{bmatrix} \tag{2.16}$$

and

$$\tau_0 = \begin{bmatrix} 1 & -\sigma \\ \sigma & -1 \end{bmatrix}. \tag{2.17}$$

Equation (1.3) with (2.14) is derived from the Lagrangian density

$$\hat{\mathscr{L}}^0 = \bar{\psi}(x)^\mu\Lambda(\partial)^{\mu\nu}\psi(x)^\nu. \tag{2.18}$$

The Lagrangian in (2.18) leads us to the Hamiltonian

$$\hat{H}^0(t) = \int \mathrm{d}^3x[\bar{\psi}^\mu\{\omega - \mathrm{i}P\}^{\mu\nu}\psi^\nu + c\text{-number}]. \tag{2.19}$$

The last c-number term in $\hat{H}^0(t)$ is irrelevant to the canonical equations, but is necessary to preserve the tilde property of $\hat{H}^0$:

$$[\mathrm{i}\hat{H}^0(t)]^\sim = \mathrm{i}H^0(t). \tag{2.20}$$

The Feynman propagator of ψ^μ and $\bar{\psi}^\mu$ is explicitly given by

$$\begin{aligned}\Delta(x_1,x_2)^{\mu\nu} &= -\mathrm{i}\langle 0|T[\psi(x_1)^\mu\bar{\psi}(x_2)^\nu]|0\rangle \\ &= \int \frac{\mathrm{d}^3k}{(2\pi)^2}\exp[\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)]B^{-1}(t_1,\boldsymbol{k})^{\mu\sigma} \\ &\quad\times\begin{bmatrix} -\mathrm{i}\theta(t_1-t_2)E_1(t_1,t_2;\boldsymbol{k}) & 0 \\ 0 & \mathrm{i}\theta(t_2-t_1)E_2(t_1,t_2;\boldsymbol{k})\end{bmatrix}^{\sigma\lambda} B(t_2,\boldsymbol{k})^{\lambda\nu}.\end{aligned} \tag{2.21}$$

Here E_μ is defined by

$$E_\mu(t_1, t_2) = E(t_1)^{\mu\mu} E^{-1}(t_2)^{\mu\mu} \qquad (\mu \text{ is not summed})$$

$$= \exp\left(-\mathrm{i}\int_{t_2}^{t_1} [\omega(s) \mp \mathrm{i}\kappa(s)]\mathrm{d}s\right) \qquad (\text{for } \mu = \tfrac{1}{2}) \tag{2.22}$$

$E^{\mu\nu}$ being the elements of the matrix E in (2.9).

Let us consider the Lagrangian for the Heisenberg field with the form:

$$\mathscr{L}_\mathrm{H} = \psi_\mathrm{H}^\dagger\left(\mathrm{i}\frac{\partial}{\partial t} - \omega_0\right)\psi_\mathrm{H} - V(\psi_\mathrm{H}, \psi_\mathrm{H}^\dagger) \tag{2.23}$$

which completely governs the behaviour law of a given dynamical system. According to the general prescription for obtaining the total Hamiltonian, $\hat{H}_\mathrm{H}$, in TFD [1, 3, 4], we have

$$\hat{H}_\mathrm{H} = H_\mathrm{H} - \tilde{H}_\mathrm{H} \tag{2.24}$$

where H_H is derived from $\mathscr{L}_\mathrm{H}$, and $\tilde{H}_\mathrm{H}$ is its tilde conjugate. We now go to the interaction representation associated with the quasiparticle free field ψ. The $\hat{H}_\mathrm{H}$ with ψ_H and $\bar{\psi}_\mathrm{H}$ being replaced by ψ and $\bar{\psi}$, respectively, will be denoted by $\hat{H}$. Then $\hat{H} = H - \tilde{H}$. The interaction Hamiltonian (see (1.4)) is then given by

$$\begin{aligned}\hat{H}_\mathrm{int}(t) &= \hat{H}(t) - \hat{H}^0(t)\\ &= \int \mathrm{d}^3x\{V(\psi^1, \bar{\psi}^1) - V(-\sigma\bar{\psi}^2, \psi^2)\\ &\quad + \bar{\psi}^\mu[-\delta\omega(t) + \mathrm{i}P(t)]^{\mu\nu}\psi^\nu\}\end{aligned} \tag{2.25}$$

with

$$-\delta\omega(t, \boldsymbol{k}) = \omega_0(\boldsymbol{k}) - \omega(t, \boldsymbol{k}). \tag{2.26}$$

In (2.25), the last term represents the counterterm. (There the c-number counterterm coming from $\hat{H}^0$ in (2.19) was suppressed since it does not have any relevance in the following calculations.)

The choice [3], (2.11), for the matrix B simplifies our computations because it leads to

$$\langle 0|\hat{H}(t) = 0 \tag{2.27}$$

and

$$\langle 0|\hat{H}^0(t) = 0. \tag{2.28}$$

Finally, from (2.25), (2.27) and (2.28), we obtain

$$\langle 0|\hat{H}_\mathrm{int}(t) = 0 \tag{2.29}$$

which in its turn gives

$$\langle 0|\hat{V}(t, t_0) = \langle 0|\hat{S} = \langle 0| \tag{2.30}$$

where $\hat{S}$ is defined in (1.5).

3. The renormalisation condition

The dynamical changes of state vectors in TFD are drastically different from those in the usual field theory.

In the latter case, the state vector space is constructed by the asymptotic (incoming or outgoing) fields which are the quasiparticle fields. These quasiparticles, as well as the vacuum state, are assumed to be stable:

$$S^{-1}|0\rangle = |0\rangle \tag{3.1a}$$

$$\langle 0|S = \langle 0| \tag{3.1b}$$

$$S^{-1}|1\rangle = |1\rangle \tag{3.1c}$$

$$\langle 1|S = \langle 1| \tag{3.1d}$$

$|1\rangle$ and S being an asymptotic single-particle state and the S-matrix operator.

The quasiparticles fields with thermal degree of freedom cannot be the asymptotic fields. This can be seen from the presence of the κ term in $\hat{H}^0$ in (2.19). This is because the spectrum of eigenvalues of $\hat{H}^0$ is not lower bounded but extends from $-\infty$ to ∞. Though our choice, (2.11), of B led to $\langle 0|\hat{S} = \langle 0|$ (see (2.30)) in TFD, we have

$$\hat{S}^{-1}|0\rangle \neq |0\rangle. \tag{3.2}$$

(Note that (2.30) does not give $\hat{S}^{-1}|0\rangle = |0\rangle$ since $\hat{S}$ is not unitary in TFD.) Furthermore, the dissipation makes almost all of the quasiparticles unstable.

The renormalisation condition provides us with a self-consistent way of establishing a definite quasiparticle picture. This is a condition applied to the transition matrix element of a Heisenberg field operator between the vacuum and a single quasiparticle state. A time-dependent non-equilibrium system usually approaches an equilibrium state at $t = \infty$. Then, it may be that the quasiparticle may be measured as a single particle precisely at the equilibrium limit. (Before the system reaching the equilibrium, the meaning of the measurements is still not clear to us.) This naturally leads us to introduce the final-state quasiparticle operators ξ_{f}^{μ} and $\bar{\xi}_{\mathrm{f}}^{\mu}$

$$\xi_{\mathrm{f}}^{\mu} = \hat{S}^{-1}\xi^{\mu}\hat{S} \tag{3.3a}$$

$$\bar{\xi}_{\mathrm{f}}^{\mu} = \hat{S}^{-1}\bar{\xi}^{\mu}\hat{S}. \tag{3.3b}$$

The renormalisation condition is to be applied to the matrix elements $\langle 1_{\mathrm{f}}(\tilde{1}_{\mathrm{f}})|\psi_{\mathrm{H}}^{\mu}|0\rangle$ and $\langle 1_{\mathrm{f}}(\tilde{1}_{\mathrm{f}})\bar{\psi}_{\mathrm{H}}^{\mu}|0\rangle$. Using (3.3), we can rewrite these matrix elements as

$$\langle 1_{\mathrm{f}}(\tilde{1}_{\mathrm{f}})|\psi_{\mathrm{H}}(x)^{\mu}|0\rangle = \langle 1(\tilde{1})|\hat{S}\psi_{\mathrm{H}}(x)^{\mu}|0\rangle \tag{3.4a}$$

$$\langle 1_{\mathrm{f}}(\tilde{1}_{\mathrm{f}})|\bar{\psi}_{\mathrm{H}}(x)^{\mu}|0\rangle = \langle 1(\tilde{1})|\hat{S}\bar{\psi}_{\mathrm{H}}(x)^{\mu}|0\rangle. \tag{3.4b}$$

Here $\langle 1_{\mathrm{f}}(\tilde{1}_{\mathrm{f}})|$ and $\langle 1(\tilde{1})|$ stand for $\langle 0|\xi_{\mathrm{f}}^{1}(\bar{\xi}_{\mathrm{f}}^{2})$ and $\langle 0|\xi^{1}(\bar{\xi}^{2})$, respectively. We are thus interested in the dynamical map of $\hat{S}\psi_{\mathrm{H}}^{\mu}$ in (1.6) rather than the one of ψ_{H}^{μ} itself. This argument is supplemented by the remark made in § 2 that only the T-product Green functions are not troubled by the divergencies caused by $\exp(-\kappa t)$ factor but are convergent. The dynamical map of ψ_{H}^{μ} carries the retarded Green functions as the expansion coefficients, as is seen from the relation

$$\psi_{\mathrm{H}}(x)^{\mu} = \hat{V}^{-1}(t, -\infty)\psi(x)^{\mu}\hat{V}(t, -\infty). \tag{3.5}$$

On the other hand, the expansion coefficients in the dynamical map of $\hat{S}\psi_{\mathrm{H}}^{\mu}$ are the T-product Green functions:

$$\hat{V}(T, -\infty)\psi_{\mathrm{H}}(x)^{\mu} = T[\hat{V}(T, -\infty)\psi(x)^{\mu}] \tag{3.6}$$

where $T > t$. For (3.6) to be valid for any t, T must be infinity. The same argument is applied to the dynamical map of $\hat{S}\bar{\psi}_{\mathrm{H}}^{\mu}$. Thus we get the combinations of $\hat{S}\psi(x)^{\mu}$ and $\hat{S}\bar{\psi}(x)^{\mu}$.

To simplify our derivation of the renormalisation condition, we consider a model which preserves the symmetry under the following global phase transformation:

$$\psi_{\mathrm{H}}^{\mu} \to \mathrm{e}^{\mathrm{i}\alpha} \psi_{\mathrm{H}}^{\mu} \tag{3.7a}$$

$$\bar{\psi}_{\mathrm{H}}^{\mu} \to \mathrm{e}^{-\mathrm{i}\alpha} \bar{\psi}_{\mathrm{H}}^{\mu}. \tag{3.7b}$$

It has already been assumed that there is no breakdown of this symmetry in the expressions for the dynamical maps (1.6). In such a symmetric case, two of four matrix elements in (3.4) vanish identically, and the other two, i.e. $\langle \tilde{1}|\hat{S}\psi_{\mathrm{H}}^{\mu}|0\rangle$ and $\langle 1\hat{S}\bar{\psi}_{\mathrm{H}}^{\mu}|0\rangle$, remain non-trivial. It follows immediately from the dynamical map (1.6) that these two matrix elements satisfy the relations:

$$\Lambda(t, \partial)^{\mu\nu}\langle \tilde{1}|\hat{S}\psi_{\mathrm{rH}}(x)^{\mu}|0\rangle = 0 \tag{3.8a}$$

$$\langle 1|\hat{S}\bar{\psi}_{\mathrm{rH}}(x)^{\nu}|0\rangle\Lambda(t, -\bar{\partial})^{\nu\mu} = 0 \tag{3.8b}$$

where $\Lambda^{\mu\nu}$ is given in (2.14) and the fields ψ_{rH}^{μ} and $\bar{\psi}_{\mathrm{rH}}^{\mu}$, called the renormalised Heisenberg fields, are defined as

$$\psi_{\mathrm{rH}}(x)^{\mu} = Z^{-1}\left(t, \frac{1}{\mathrm{i}}\vec{\nabla}\right)^{\mu\nu}\psi_{\mathrm{H}}(x)^{\nu} \tag{3.9a}$$

$$\bar{\psi}_{\mathrm{rH}}(x)^{\mu} = \bar{\psi}_{\mathrm{H}}(x)^{\nu}\bar{Z}^{-1}\left(t, -\frac{1}{\mathrm{i}}\bar{\vec{\nabla}}\right)^{\nu\mu} \tag{3.9b}$$

respectively (see (1.6) for Z and $\bar{Z}$). The relations in (3.8) are called the renormalisation conditions.

Since $\hat{V}(t, -\infty) = 1$ at $t = -\infty$, (3.5) gives the initial conditions

$$\psi_{\mathrm{H}}(x)^{\mu} = \psi(x)^{\mu} \qquad \text{at } t = -\infty \tag{3.10a}$$

$$\bar{\psi}_{\mathrm{H}}(x)^{\mu} = \bar{\psi}(x)^{\mu} \qquad \text{at } t = -\infty. \tag{3.10b}$$

In the usual quantum field theory, the self-energy loop diagrams create $(Z-1)\psi$, giving rise to the $Z\psi$ term in the dynamical map. (Another way of formulation is to use the weak limit which gives $\psi_{\mathrm{H}} \to Z\psi$ at $t = -\infty$.) To avoid this complication we formulate the entire theory in terms of ψ_{rH} and $\bar{\psi}_{\mathrm{rH}}$ (instead of ψ_{H} and $\bar{\psi}_{\mathrm{H}}$) so that we have

$$\psi_{\mathrm{rH}}(x)^{\mu} = \psi(x)^{\mu} \qquad \text{at } t = -\infty \tag{3.11a}$$

$$\bar{\psi}_{\mathrm{rH}}(x)^{\mu} = \bar{\psi}(x)^{\mu} \qquad \text{at } t = -\infty. \tag{3.11b}$$

Then $\hat{V}$ is naturally made to satisfy

$$\psi_{\mathrm{rH}}(x)^{\mu} = \hat{V}(t, -\infty)\psi(x)^{\mu}\hat{V}(t, -\infty) \tag{3.12}$$

etc, with $\hat{V}(t, -\infty) = 1$ at $t = -\infty$ instead of (3.5). To achieve this, we replace ψ_{H} and $\bar{\psi}_{\mathrm{H}}$ by $Z\psi_{\mathrm{rH}}$ and $\bar{\psi}_{\mathrm{H}}\bar{Z}$ in the Lagrangian. Then, the interaction Lagrangian $\mathscr{L}_{\mathrm{int}}$ is given by

$$\begin{aligned}\hat{\mathscr{L}}_{\mathrm{int}}(x) = {} & V((Z\psi)^{1}, (\bar{\psi}\bar{Z})^{1}) - V(-\sigma(\bar{\psi}\bar{Z})^{2}, (Z\psi)^{2}) \\ & + \bar{\psi}^{\mu}\left\{\bar{Z}\mathrm{i}\frac{\partial}{\partial t}Z - \mathrm{i}\frac{\partial}{\partial t} - \bar{Z}Z\omega_{0} + \omega - \mathrm{i}P\right\}^{\mu\nu}\psi^{\nu}.\end{aligned} \tag{3.13}$$

$\hat{V}$ is given by (1.4) with $\hat{H}_{\mathrm{int}} = -\hat{\mathscr{L}}_{\mathrm{int}}$. In particular, $\hat{S}$ is expressed as

$$\hat{S} = T\left[\exp\left(\mathrm{i}\int_{-\infty}^{\infty}\mathscr{L}_{\mathrm{int}}\,\mathrm{d}^{4}x\right)\right]. \tag{3.14}$$

With this redefinition of $\hat{H}_{\mathrm{int}}$, the whole of the previous argument remains unchanged.

4. An analysis of the renormalisation condition

In this section we rewrite the conditions (3.8), putting them into a more manageable form, and derive from them a set of equations for a set of physical parameters.

The matrices Z and $\bar{Z}$ in (3.13), as a result of their appearance in (1.6), introduce new degrees of freedom in addition to the parameters ω, κ and n. In the first place, we see that Z and $\bar{Z}$ are not independent because the tilde conjugation rules in (2.7) yield a constraint

$$\bar{Z} = \tau_2 Z^\dagger \tau_2. \tag{4.1}$$

In deriving (4.1), it was considered that (2.7) holds true both for the quasiparticle field and the Heisenberg field. (This gives $[\hat{V}(t, -\infty)]^{\sim} = \hat{V}(t, -\infty)$.) We put Z in the form

$$Z = \exp\left[\mathrm{i}\chi\left(t, \frac{1}{\mathrm{i}}\vec{\nabla}\right)\tau_3\right]\rho_Z \tag{4.2}$$

where χ is a real parameter. Since the following approximation needs only the freedom in choice of χ, we choose approximately $\rho_Z = 1$ in this paper. This leads to the simple relation

$$\bar{Z} = Z = \exp(\mathrm{i}\chi\tau_3). \tag{4.3}$$

The freedom in choice of ρ_Z corresponds to the wavefunction renormalisation constant in the ordinary field theory.

With the definition $\langle\tilde{1}| = \langle 0|\xi_k^2$ and the relation $\langle 0|\xi_k^1 = 0$, it is straightforward to show that (3.8a) is equivalent to

$$E\left(t', -\frac{1}{\mathrm{i}}\vec{\nabla}'\right)^{22}\Lambda(t,\partial)^{\mu\sigma}\langle 0|\bar{\psi}(x')^\nu \hat{S}\psi_{r\mathrm{H}}(x)^\sigma|0\rangle = 0 \tag{4.4}$$

for any t'. Here use was made of (2.1b), (2.9) and (2.13b). By taking sufficiently large t', we can rewrite it as

$$\lim_{t'\to\infty} E\left(t', -\frac{1}{\mathrm{i}}\vec{\nabla}'\right)^{22}\Lambda(t,\partial)^{\mu\sigma}G(x,x')^{\sigma\nu} = 0 \tag{4.5}$$

where G is the time-ordered full propagator:

$$G(x,x')^{\mu\nu} = -\mathrm{i}\langle 0|T[\hat{S}\psi(x)^\mu\bar{\psi}(x')^\nu]|0\rangle \tag{4.6}$$

with the expressions (3.14) for $\hat{S}$ and (4.3) for Z and $\bar{Z}$. Since G satisfies the Dyson equation:

$$G(x,x')^{\mu\nu} = \Delta(x,x')^{\mu\nu} + \int \mathrm{d}y_1\,\mathrm{d}y_2\,\Delta(x,y_1)^{\mu\sigma}\Sigma(y_1,y_2)^{\sigma\lambda}G(y_2,x')^{\lambda\nu} \tag{4.7}$$

where Δ is found in (2.21) satisfying

$$\Lambda(t,\partial)^{\mu\sigma}\Delta(x,x')^{\sigma\nu} = \delta(x-x')\delta^{\mu\nu} \tag{4.8}$$

and Σ is the 2×2 matrix self-energy. Equation (4.5) is now

$$0 = \lim_{t'\to\infty}\int \mathrm{d}y\,\Sigma(x,y)^{\mu\sigma}\Delta(y,x')^{\sigma\nu}E(t')^{22} \tag{4.9}$$

$$= \int \mathrm{d}y\,\Sigma(x,y)^{\mu\sigma}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{\mathrm{i}k\cdot(y-x')}\mathrm{i}E(s)^{22}\left[B^{-1}(s)\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}B(t')\right]^{\sigma\nu} \tag{4.10}$$

after the substitution of (2.21). Here t and s are the time components of x and y, respectively. In (4.9) and hereafter, we suppress the momentum symbols, i.e. $\vec{\nabla}/\mathrm{i}$ or

$\boldsymbol{k}$, until they are needed. The self-energy Σ is a sum of the loop contribution $\Sigma_{\rm L}$ and the counterterm $\Sigma_{\rm c}$:

$$\Sigma(x, x') = \Sigma_{\rm L}(x, x') + \Sigma_{\rm c}(x, x'). \tag{4.11}$$

According to (3.13), we have

$$\Sigma_{\rm c}(x, x')^{\mu\nu} = \left(-(\bar{Z}Z - 1)\mathrm{i}\frac{\partial}{\partial t} + (\bar{Z}Z\omega_0 - \omega) + \mathrm{i}P - \mathrm{i}\bar{Z}\dot{Z} \right)^{\mu\nu} \delta(x - x'). \tag{4.12}$$

Since the self-energy $\Sigma_{\rm c}$ is operating on Δ from the left in (4.9), we can replace $\Sigma_{\rm c}$ with $\Sigma_{\rm c}^{\rm eff}$ considering (4.8):

$$\Sigma_{\rm c}^{\rm eff}(x, x')^{\mu\nu} = \{\exp[2\mathrm{i}\chi(t)\tau_3][\omega_0 - \omega(t) + \mathrm{i}P(t) + \dot{\chi}(t)\tau_3]\}^{\mu\nu}\delta(x - x') \tag{4.13}$$

where (4.3) was made use of.

Several matrix formulae are listed below to manipulate (4.9) or (4.10). Defining the matrices

$$T_1 = \begin{bmatrix} 1 & 0 \\ \sigma & 0 \end{bmatrix} \tag{4.14a}$$

$$T_2 = \begin{bmatrix} 0 & 0 \\ -\sigma & 1 \end{bmatrix} \tag{4.14b}$$

we write

$$B^{-1}(s)\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} B(t) = T_2 - \sigma n(s)\tau_0 \tag{4.15a}$$

$$A(t) = T_1 - T_2 + 2\sigma n(t)\tau_0 \tag{4.15b}$$

where $A(t)$ and τ_0 are seen in (2.16) and (2.17), respectively. The matrices T_1, T_2 and τ_0 form a closed algebra:

$$T_1^2 = T_1 \qquad T_2^2 = T_2 \tag{4.16a}$$

$$\tau_0^2 = T_1T_2 = T_2T_1 = \tau_0T_1 = T_2\tau_0 = 0 \tag{4.16b}$$

$$T_1\tau_0 = \tau_0T_2 = \tau_0. \tag{4.16c}$$

Using these relations together with

$$\tau_3T_2 = -T_2 \tag{4.17a}$$

$$\tau_3\tau_0 = 2T_2 + \tau_0 \tag{4.17b}$$

$$\exp(2\mathrm{i}\chi\tau_3)T_2 = \exp(-2\mathrm{i}\chi)T_2 \tag{4.17c}$$

$$\exp(2\mathrm{i}\chi\tau_3)\tau_0 = \exp(2\mathrm{i}\chi)\tau_0 + [\exp(2\mathrm{i}\chi) - \exp(-2\mathrm{i}\chi)]T_2 \tag{4.17d}$$

we obtain

$$\begin{aligned}\int \mathrm{d}y\, \Sigma_{\rm c}(x, y)^{\mu\sigma} \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{y}-\boldsymbol{x}')}\mathrm{i}E(s)^{22}\left[B^{-1}(s)\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}B(t)\right]^{\sigma\nu} \\ = \int \frac{d^3k}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\mathrm{i}E(t)^{22}[\exp[-2\mathrm{i}\chi(t)]\{[1 + \sigma n(t)][\omega_0 - \omega(t) - \dot{\chi}(t)] \\ - \mathrm{i}\{[1 + \sigma n(t)]\kappa(t) + \sigma\dot{n}(t)\}\}T_2 \\ + \exp[2\mathrm{i}\chi(t)]\{-\sigma n(t)[\omega_0 - \omega(t) + \dot{\chi}(t)] \\ + \mathrm{i}[\sigma n(t)\kappa(t) + \sigma\dot{n}(t)]\}(T_2 + \tau_0)]^{\mu\nu}\end{aligned} \tag{4.18}$$

and

$$\int \mathrm{d}y\, \Sigma_{\mathrm{L}}(x,y)^{\mu\sigma} \mathrm{i}E(s)^{22}\left[B^{-1}(s)\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}B(t)\right]^{\sigma\nu}$$
$$= \int \mathrm{d}y \int \frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{y}-\boldsymbol{x}')}\mathrm{i}E(s)^{22}\{[\Sigma_{\mathrm{L}}(t,s)^{22}+n(s)(\Sigma_{\mathrm{L}}(t,s)^{21}+\sigma\Sigma_{\mathrm{L}}(t,s)^{22})]T_2$$
$$+[-\sigma\Sigma_{\mathrm{L}}(t,s)^{12}-\sigma n(s)(\Sigma_{\mathrm{L}}(t,s)^{11}+\sigma\Sigma_{\mathrm{L}}(t,s)^{12})](\tau_0+T_2)\}^{\mu\nu}. \qquad (4.19)$$

When (4.18) and (4.19) are used, the renormalisations condition (4.10) yields the following four *independent* real equations after a three-dimensional Fourier transformation is performed:

$$\omega_0(\boldsymbol{k})-\omega(t,\boldsymbol{k})-\dot{\chi}(t,\boldsymbol{k})=\frac{-\mathrm{Re}\, I(t,\boldsymbol{k})^2}{1+\sigma n(t,\boldsymbol{k})} \qquad (4.20a)$$

$$\omega_0(\boldsymbol{k})-\omega(t,\boldsymbol{k})+\dot{\chi}(t,\boldsymbol{k})=\frac{-\mathrm{Re}\, I(t,\boldsymbol{k})^1}{n(t,\boldsymbol{k})} \qquad (4.20b)$$

$$\mathrm{i}\{[1+2\sigma n(t,\boldsymbol{k})]\kappa(t,\boldsymbol{k})+\sigma\dot{n}(t,\boldsymbol{k})\}=\mathrm{Im}\, I(t,\boldsymbol{k})^2+\mathrm{i}\sigma n(t,\boldsymbol{k})\kappa(t,\boldsymbol{k}) \qquad (4.20c)$$

$$\mathrm{i}[2n(t,\boldsymbol{k})\kappa(t,\boldsymbol{k})+\dot{n}(t,\boldsymbol{k})]=\mathrm{Im}\, I(t,\boldsymbol{k})^1+\mathrm{i}n(t,\boldsymbol{k})\kappa(t,\boldsymbol{k}) \qquad (4.20d)$$

where

$$I(t,\boldsymbol{k})^{\alpha}=\exp[(-1)^{\alpha}2\mathrm{i}\chi(t,\boldsymbol{k})]\int \mathrm{d}s\, E_2(s,t;\boldsymbol{k})$$
$$\times\{\Sigma_{\mathrm{L}}(t,s;\boldsymbol{k})^{\alpha 2}+n(s,\boldsymbol{k})[\Sigma_{\mathrm{L}}(t,s;\boldsymbol{k})^{\alpha 1}+\sigma\Sigma_{\mathrm{L}}(t,s)^{\alpha 2}]\} \qquad (\alpha=1,2) \quad (4.21)$$

E_2 being given in (2.22) and $\Sigma_{\mathrm{L}}(t,s;\boldsymbol{k})^{\mu\nu}$ defined by

$$\Sigma_{\mathrm{L}}(x,y)^{\mu\nu}=\int\frac{\mathrm{d}^3k}{(2\pi)^3}\exp[\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})]\Sigma_{\mathrm{L}}(t,s;\boldsymbol{k})^{\mu\nu}. \qquad (4.22)$$

We write (4.20) in such a manner that the left-hand sides and the right-hand sides come exclusively from the counterterms and the propagators, respectively. The equations in (4.20) form a closed set of equations to determine four parameters ω, κ, n and χ, while the right-hand sides of (4.20) are functionals of these parameters. Thus, these are the self-consistent equations on ω, κ, n and χ which follow from the renormalisation condition.

We give an example of the calculation to show more clearly what (4.20) imply, using a model [4, 7] of a fermion field ψ and a boson field ϕ (both are of type 1) interacting with each other through the Yukawa-type interaction, $g\psi^{\dagger}\psi\phi+\mathrm{HC}$. Quantities associated with ψ and ϕ will be distinguished by the subscripts F and B, respectively. In this case, $\hat{\mathscr{L}}_{\mathrm{int}}$ in (3.13) has the following explicit form:

$$\mathscr{L}_{\mathrm{int}}(x)=g\{[\exp(\mathrm{i}\chi_F)\bar{\psi}^1][\exp(\mathrm{i}\chi_F)\psi^1][\exp(\mathrm{i}\chi_B)(\phi^1+\bar{\phi}^1)]$$
$$-[\exp(-\mathrm{i}\chi_F)\psi^2][\exp(-\mathrm{i}\chi_F)\bar{\psi}^2][\exp(-\mathrm{i}\chi_B)(\phi^2-\bar{\phi}^2)]\}$$
$$+\bar{\psi}^{\mu}\left([\exp(2\mathrm{i}\chi_F\tau_3)-1]\mathrm{i}\frac{\partial}{\partial t}-\exp(2\mathrm{i}\chi_F\tau_3)(\omega_{F0}+\dot{\chi}_F\tau_3)+\omega_F-\mathrm{i}P_F\right)^{\mu\nu}\psi^{\nu}$$
$$+\bar{\phi}^{\mu}\left([\exp(2\mathrm{i}\chi_B\tau_3)-1]\mathrm{i}\frac{\partial}{\partial t}-\exp(2\mathrm{i}\chi_B\tau_3)(\omega_{b0}+\dot{\chi}_B\tau_3)+\omega_B-\mathrm{i}P_B\right)^{\mu\nu}\phi^{\nu}. \qquad (4.23)$$

When only the one-loop diagrams is picked up, the self-energy for ψ due to the loop contribution is obtained as

$$\Sigma_{FL}(t, s; \boldsymbol{k})^{\mu\nu} = \Sigma_{+FL}(t, s; \boldsymbol{k})^{\mu\nu} + \Sigma_{-FL}(t, s; \boldsymbol{k})^{\mu\nu} \tag{4.24}$$

with

$$\begin{aligned}\Sigma_{+FL}(t, s; \boldsymbol{k})^{\mu\nu} = \mathrm{i}g^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \Bigg(&\exp\{\mathrm{i}[\chi_F(t, \boldsymbol{k}) + \chi_F(t, \boldsymbol{k}-\boldsymbol{q}) + \chi_B(t, \boldsymbol{q})]\tau_3\} \\ &\times \begin{bmatrix} \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{11}\Delta_B(t, s; \boldsymbol{q})^{11} & -\Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{12}\Delta_B(t, s; \boldsymbol{q})^{12} \\ \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{21}\Delta_B(t, s; \boldsymbol{q})^{21} & -\Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{22}\Delta_B(t, s; \boldsymbol{q})^{22} \end{bmatrix} \\ &\times \exp\{\mathrm{i}[\chi_F(s, \boldsymbol{k}) + \chi_F(s, \boldsymbol{k}-\boldsymbol{q}) + \chi_B(s, \boldsymbol{q})]\tau_3\} \Bigg)^{\mu\nu} \end{aligned} \tag{4.25a}$$

$$\begin{aligned}\Sigma_{-FL}(t, s; \boldsymbol{k})^{\mu\nu} \\ = \mathrm{i}g^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \Bigg(&\exp\{\mathrm{i}[\chi_F(t, \boldsymbol{k}) + \chi_F(t, \boldsymbol{k}-\boldsymbol{q}) + \chi_B(t, \boldsymbol{q})]\tau_3\} \\ &\times \begin{bmatrix} \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{11}\Delta_B(s, t; \boldsymbol{q})^{11} & \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{12}\Delta_B(s, t; \boldsymbol{q})^{21} \\ -\Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{21}\Delta_B(s, t; \boldsymbol{q})^{12} & -\Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{22}\Delta_B(s, t; \boldsymbol{q})^{22} \end{bmatrix} \\ &\times \exp\{\mathrm{i}[\chi_F(s, \boldsymbol{k}) + \chi_F(s, \boldsymbol{k}-\boldsymbol{q}) + \chi_B(s, \boldsymbol{q})]\tau_3\} \Bigg)^{\mu\nu} \end{aligned} \tag{4.25b}$$

where $\Delta(t, s; \boldsymbol{k})^{\mu\nu}$ is defined in (2.21). The expression (4.24) with (4.25) is put into (4.21), which appears in the right-hand side of (4.20). For example, (4.20a) is now explicitly

$$\begin{aligned} \omega_0(\boldsymbol{k}) - \omega(t, \boldsymbol{k}) - \dot{\chi}(t, \boldsymbol{k}) \\ = \mathrm{i}g^2\, \mathrm{Im}\Bigg(\int \mathrm{d}s\, E_2(t, \boldsymbol{k}) &\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \exp\{\mathrm{i}[\chi_F(t, \boldsymbol{k}) - \chi_F(t, \boldsymbol{k}-\boldsymbol{q}) - \chi_B(\boldsymbol{q})]\} \\ &\times ([1 - n_F(s, \boldsymbol{k})] \exp\{-\mathrm{i}[\chi_F(s, \boldsymbol{k}) + \chi_F(s, \boldsymbol{k}-\boldsymbol{q}) + \chi_F(s, \boldsymbol{q})]\} \\ &\times \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{22}[\Delta_B(t, s; \boldsymbol{q})^{22} + \Delta_B(s, t; \boldsymbol{q})^{22} \\ &+ n_F(s, \boldsymbol{k}) \exp\{-\mathrm{i}[\chi_F(s, \boldsymbol{k}) + \chi_F(s, \boldsymbol{k}-\boldsymbol{q}) + \chi_F(s, \boldsymbol{q})]\} \\ &\times \Delta_F(t, s; \boldsymbol{k}-\boldsymbol{q})^{21}[\Delta_B(t, s; \boldsymbol{q})^{21} - \Delta_B(s, t; \boldsymbol{q})^{12}]) \Bigg) [1 - n_F(t, \boldsymbol{k})]^{-1}. \end{aligned} \tag{4.26}$$

Similarly, the calculation for ϕ can be done. Eventually we have eight simultaneous differential-integral equations for the parameters $(\omega_F, \kappa_F, n_F, \chi_F)$ and $(\omega_B, \kappa_B, n_B, \chi_B)$.

5. Conclusions

In this paper we have presented a universal expression of the renormalisation condition (3.8) in time-dependent thermal situations of quantum field systems. The concept of quasiparticles in thermal doublet space played a crucial role in achieving it. The renormalisation condition thus obtained provides us with a closed set of four independent self-consistent equations (4.20) determining four real parameters ω, κ, n and χ, which are generally functions of t and $\boldsymbol{k}$. The introduction of the new degree of freedom χ is indispensable for consistency of the renormalisation condition. The χ degree of freedom naturally appears in the dynamical map (i.e. in the language of quantum field theory) as we have seen, but it seems very difficult to identify it in the density matrix formalism (i.e. in the language of statistical mechanics).

We gave the discussion only on the renormalisation condition corresponding to the energy (or mass) renormalisation in the usual quantum field theory, by choosing $\rho_z = 1$ in (4.2). Another condition, corresponding to the wavefunction renormalisation which fixes the matrix ρ_z, should be studied to complete the formalism. This problem still remains open.

Equations (4.20) are very complicated to solve as they are simultaneous differential-integral ones. One systematic method of approximation to solve them is provided by a successive one [4, 7] in which all the parameters are expanded around those at final equilibrium state:

$$\omega(t) = \omega_0 + \Delta\omega_1(t) + \Delta\omega_2(t) + \ldots \tag{5.1a}$$

$$\kappa(t) = \kappa_0 + \Delta\kappa_1(t) + \Delta\kappa_2(t) + \ldots \tag{5.1b}$$

$$n(t) = n_0 + \Delta n_1(t) + \Delta n_2(t) + \ldots \tag{5.1c}$$

$$\chi(t) = \chi_0 + \Delta\chi_1(t) + \Delta\chi_2(t) + \ldots . \tag{5.1d}$$

The subscript represents the power of coupling in the sense of perturbation calculation. When one calculates $\Delta\omega_m$, etc, one uses the propagators with the parameters of order less than m. The iteration of this process improves the approximation order by order.

As shown in [4, 7], we exploited the successive approximation at one-loop level for a self-interacting quantum field system. There we *ad hoc* assumed the following renormalisation conditions on the self-energy:

$$\Sigma(t, k_0 = \omega) = \Sigma(k_0 = \omega, t) = 0 \tag{5.2}$$

for any t, where

$$\Sigma(t, k_0) = \int \frac{\mathrm{d}s}{2\pi} \exp(-\mathrm{i}k_0 s)\Sigma(t, s) \tag{5.3a}$$

$$\Sigma(k_0, t) = \int \frac{\mathrm{d}s}{2\pi} \exp(\mathrm{i}k_0 s)\Sigma(s, t) \tag{5.3b}$$

and obtained the expressions for ω and κ in this order and the master equation:

$$\dot{n}(t) + 2\kappa[n(t) - N_0] = 0 \tag{5.4}$$

where N_0 is constant. Recall that the parameter χ is not taken account of in [4, 7].

Now we can repeat the same approximation for the renormalisation conditions (4.20), implying that the parameters on the right-hand side is of the zeroth order, particularly $\chi_0 = \dot{\chi}_0 = 0$ while those on the left-hand side is of the first one. From straightforward calculation, it turns out that the same result for ω, κ and the master equation in (5.4) as in the preceding paragraph from (5.2) is derived as long as $\chi = \dot{\chi} = 0$ holds. With (4.20) one can go to any higher orders step by step to reveal the short-time behaviour of the system, whereas (5.2) is no longer valid at the next order since then ω becomes time dependent.

The application of TFD to non-equilibrium phase transitions is quite important. In this case, the temporal change of the order parameter naturally makes ω time dependent. For this reason, the universal condition (4.20) has been looked for. The study of phase transition is under progress when (4.20) is made use of.

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Note added in proof. Our total Hamiltonian $\hat{H}$, when expressed in terms of the renormalised fields defined by (3.9) and with the choice of (4.3), no longer has a Hermitian outlook due to the Z factor. Then (2.27) does not hold true and neither does (2.30), i.e. $\langle 0|S \neq \langle 0|$, although none of the results in the paper are influenced by this. We only have to change the definition of $\langle 1_f(\tilde{1}_f)|$ in (3.4) to $\langle 0|\hat{S}\xi_f^1(\bar{\xi}_f^2)$ instead of $\langle 0|\xi_f^1(\bar{\xi}_f^2)$; otherwise the whole argument in this paper remains unchanged. We would like to thank Ian Hardman for pointing this out.

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